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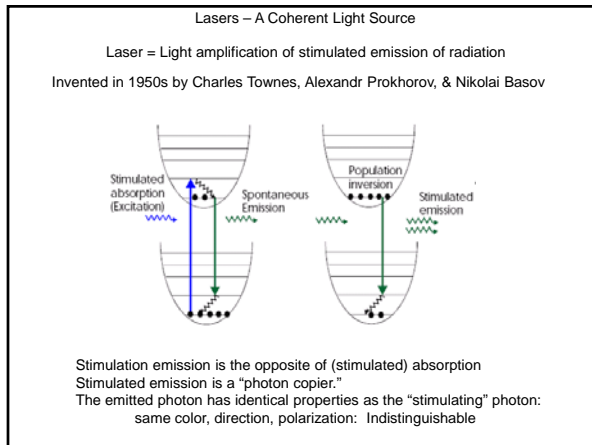
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What does all these have to do with laser?

What is good about laser? What we want from a light source that a lamp is not?

- (1) Monochromatic
- (2) High spectral radiance
- (3) Low divergence
- (4) Unique polarization
- (5) Long coherence length

How do we achieve this? If we can "copy" a photon many times. Then we have IT ... laser, Nobel prize, ....

We need two things: (1) amplification, (2) population inversion

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**Basic Idea of A Laser**

Put a lasing medium in "population inversion" between two mirrors. An avalanche, amplification, process occurs as a spontaneous emitted photon bounce back and forth between the two mirrors to "copy" more of itself.

Uni-directionality comes from the fact that only photons going between the two mirrors are amplified.

Population inversion is needed so that the probability of stimulated emission is much greater than absorption to sustained the avalanche process

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**A Better Understanding of How Light Interaction with Molecules Is Needed to Know How to Create Population Inversion**

Einstein studied these three processes: stimulated absorption, Stimulated emission, and spontaneous emission

Let  $N_i, N_j$  be the population of molecules in the ground and excited states

Let  $I$  be the energy density of light

$$\frac{dN_j}{dt} = -B_{ji}N_jI - A_{ji}N_j + B_{ij}N_iI$$

At steady state, the ground and excited states population is constant:

$$B_{ij}N_iI = B_{ji}N_jI + A_{ji}N_j \quad \frac{N_j}{N_i} = \frac{B_{ij}I}{B_{ji}I + A_{ji}}$$


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The relative populations of the states not only can be determined by kinetic consideration but also by thermodynamics

We also know from statistical mechanics that the populations of two states in thermal equilibrium is described by Boltzmann statistics

$$N_{i,j} = N_0 e^{-\frac{E_{i,j}}{kT}}$$

where  $E$  is the energy of states  $i$  and  $j$ ,  $k$  is the Boltzmann's constant and  $T$  is the absolute temperature

$$\frac{N_j}{N_i} = e^{-\frac{(E_j - E_i)}{kT}} = e^{-\frac{\Delta E}{kT}}$$


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Einstein B Coefficients

Combining kinetics and thermodynamics: 
$$\rho = \frac{\frac{A_{ji}}{B_{ji}}}{\frac{B_{ij}}{B_{ji}} e^{\frac{\Delta E}{kT}} - 1}$$

What happens when temperature approach infinity?

We expect light density also grows to infinity (driving all the molecules instantly to the excited state).

$\rho \rightarrow \infty$  as  $T \rightarrow \infty$  requires  $B_{ij} = B_{ji} = B$

"Einstein B coefficient"  
**The rate constants of stimulated absorption and emission are equal**

$$\rho = \frac{A}{B e^{\frac{\Delta E}{kT}} - 1}$$


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Population inversion is not possible in a 2-state system

Low Light                      High Light

As excited state becomes well populated, the excitation and de-excitation probability becomes equal because the Einstein coefficients for up and down are equal

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Population inversion is possible in a 3-state system

Population inversion can be created if:

- (1) Spontaneous decay rate to from state 3 to state 2 is the fastest
- (2) Stimulated excitation rate from state 1 to state 3 is faster or comparable to the decay rate from state 2 to state 1
- (3) Population inversion is created between state 1 and 2
- (4) Increasing "pump" power (orange) do not cause stimulated emission from state 2

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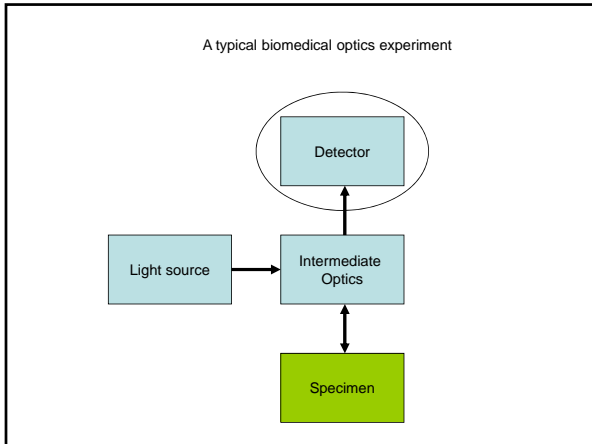
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- Noise Sources of A Detector
1. Photon Shot Noise – Counting statistics of the signal photons
  2. Dark Current Noise – Counting statistics of spontaneous electron generated in the device
  3. Johnson Noise – Thermally induced current in the transimpedance amplifier

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Optical Shot Noise

Photon arrival at detector are statistically independent, "uncorrelated", events

What do we meant by uncorrelated?

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} (n(t+\tau) - \bar{n})(n(t) - \bar{n})^* = \langle \Delta n(t+\tau) \Delta n^*(t) \rangle = 0 \quad \tau \neq 0$$

(\* denotes complex conjugate)

Although the mean number of photons arriving per unit time,  $\lambda$ , is constant on average, at each measurement time interval, the number of detected photons can vary.

The statistical fluctuation of these un-correlated random events are characterized by Poisson statistics.

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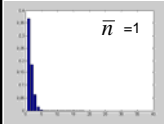
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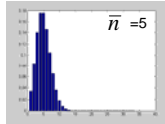
**Poisson Statistics**

If the mean number of photon detected is  $\bar{n}$ , the probability of observing  $n$  photons in time interval  $t$  is:

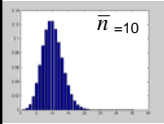
$$P(n|\bar{n}) = e^{-\bar{n}} \frac{\bar{n}^n}{n!}$$



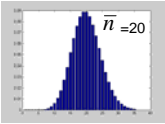
$\bar{n} = 1$



$\bar{n} = 5$



$\bar{n} = 10$



$\bar{n} = 20$

Mean:

$$\bar{n} = \frac{1}{M} \sum_i n_i$$

Variance:

$$\sigma_n^2 = \frac{1}{M} \sum_i (n_i - \bar{n})^2$$

$$\bar{n} = \sigma_n^2$$

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**Spectrum of Poisson Noise I**

$$\Delta \tilde{I}(f) = \int_{-\infty}^{\infty} \Delta I(t) e^{-i2\pi f t} dt \quad \text{where} \quad \Delta I(t) = q \Delta f (n(t) - \bar{n})$$

Assume photon number is Poisson distributed

Power spectral density:  $\tilde{P}(f) = R \Delta f \Delta \tilde{I}^*(f) \Delta \tilde{I}(f)$

Noise power:  $\tilde{N}(f, \Delta f) = \tilde{P}(f) \Delta f$

The power spectral density can be evaluated in a slightly round about way by considering the autocorrelation function:

Autocorrelation function:  $g(\tau) = R \Delta f \int_{-\infty}^{\infty} \Delta I(t+\tau) \Delta I(t) dt$

Because the event of Poisson process is completely independent of each other

$$g(\tau) = R \sigma_n^2 \delta(\tau) / \Delta f$$


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**Spectrum of Poisson Noise II**

$\delta(\tau)$  is the Dirac-Delta function with the following properties:

It has the unit of frequency

$\delta(0) = \infty; \delta(t) = 0 \text{ for } t \neq 0$

$\int \delta(t) dt = 1; \int f(t) \delta(t - \tau) dt = f(\tau)$



From Poisson process:  $\sigma_n^2 = 2\alpha q \Delta f \langle I \rangle$

Factor of 2 account for positive and negative frequency bands

The autocorrelation function of Poisson noise is:

$$g(\tau) = 2R\alpha q \langle I \rangle \delta(\tau)$$


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Spectrum of Poisson Noise III

Wiener-Khinchine Theorem:  $\tilde{P}(f) = \int g(\tau)e^{-i2\pi f\tau} d\tau$

Let's why Wiener-Khinchine theorem is true:

$$\int_{-\infty}^{\infty} g(\tau)e^{-i2\pi f\tau} d\tau = R\Delta f \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Delta I(t+\tau)\Delta I(t)dt e^{-i2\pi f\tau} d\tau$$

$$= R\Delta f \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Delta I(t+\tau)e^{-i2\pi f\tau} d\tau \Delta I(t)dt$$

$$= R\Delta f \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Delta I(\tau')e^{-i2\pi f\tau'} d\tau' e^{+i2\pi ft} \Delta I(t)dt$$

$$\tau' = t + \tau, d\tau' = d\tau$$

$$= R\Delta f \left[ \int_{-\infty}^{\infty} \Delta I(\tau')e^{-i2\pi f\tau'} d\tau' \right] \left[ \int_{-\infty}^{\infty} \Delta I(t)e^{+i2\pi ft} dt \right]$$

$$= R\Delta f \tilde{\Delta I}(f) \tilde{\Delta I}(f)^*$$

Fourier transform of the autocorrelation function is the power spectral density

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Spectrum of Poisson Noise IV

$$\tilde{P}(f) = \int_{-\infty}^{\infty} 2R\alpha q \langle I \rangle \delta(\tau) e^{-i2\pi f\tau} df = 2R\alpha q \langle I \rangle$$

Poisson noise has a "white" spectrum

Noise in a given spectral band:

$$\tilde{N}(f, \Delta f) = 2R\alpha q \langle I \rangle \Delta f$$


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Photon Shot Noise

The origin of the photon shot noise comes from the Poisson statistics of the incoming photons itself

The shot noise power is:

$$\tilde{N}_s(f, \Delta f) = 2R\alpha q \langle I \rangle \Delta f$$

The signal power is:  $S = \langle I \rangle^2 R$

$$SNR = \frac{\langle I \rangle}{2\alpha q \Delta f} = \frac{\alpha q \bar{n} / \Delta t}{2\alpha q \Delta f} = \frac{2\alpha q \bar{n} \Delta f}{2\alpha q \Delta f} = \bar{n}$$

Used sampling theorem:  $1 / \Delta t = 2\Delta f$

A detector is consider to be "ideal" if it is dominated by just shot noise.

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Dark Current Noise

The ideal photoelectric or photovoltaic device does not produce current (electrons) in the absence of light. However, thermal effect results in some probability of spontaneous production of free electrons. This effect is measured by the dark current amplitude of the device:  $\langle I_d \rangle$

The average dark current is constant at constant temperature, but the electron generated fluctuate in time according to Poisson statistic similar to the fluctuation of the signal photons.

From our discussion of photon shot noise, we have immediately

$$\tilde{N}_d(f, \Delta f) = 2R\alpha q \langle I_d \rangle \Delta f$$

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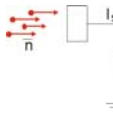
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Johnson Noise

Johnson noise originates from the temperature dependent fluctuation in the load resistance R of the transimpedance detection circuit.



Consider a simple dimensional analysis argument:

Thermal energy:  $kT$

Thermal power:  $kT\Delta f$

Power of Johnson noise current  $I_J$ :  $I_J^2 R$

$$I_J = \sqrt{\frac{kT\Delta f}{R}}$$

$$\tilde{N}_J(f, \Delta f) = kT\Delta f$$

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Characterizing Photodetectors

1. Quantum Efficiency: The probability of generating of a photoelectron from an incident photon
2. Internal Amplification: The amplification ratio for converting a photoelectron into an output current
3. Dynamic Range: What is the largest and the lowest signal that can be measured linearly
4. Response Speed: The time difference and spread between an incoming photon and the output current burst
5. Geometric form factor: Size and shape of the active area and the detector
6. Noise: Discussed extensively already

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**Photomultiplier tube (PMT)**

The PMT are characterized by two important parameters

Labels in diagram: HV photon, photocathode, e<sup>-</sup>, dynodes, anode

Cathode sensitivity,  $S$  (A/W): 0.06 A/W

Gain,  $\alpha$ :  $10^7$  to  $10^8$

We can relate current measured at the anode to the number of incident photons,  $n$ , arriving within a time interval  $\Delta t$

$$I = S \cdot \alpha \cdot E_\gamma \cdot n / \Delta t$$

$E_\gamma$  is photon energy

For green (500 nm wavelength) photons:

$$E_\gamma = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \text{ Js} \cdot 3 \times 10^8 \text{ m/s}}{5 \times 10^{-7} \text{ m}} = 4 \times 10^{-19} \text{ J}$$

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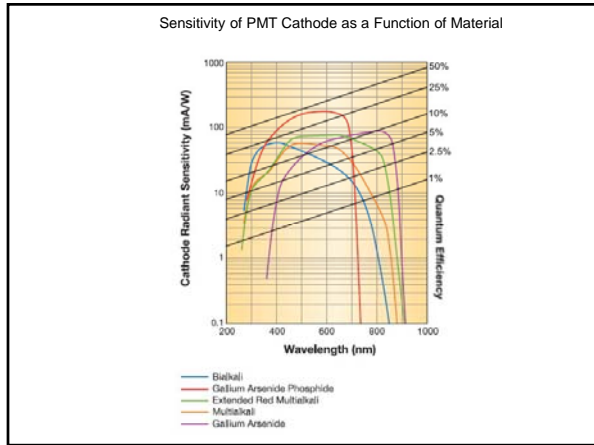
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**Photodiodes**

Labels: Anode, p-type, Depletion Region, n-type, Cathode

Biassing can increase device temporal Response speed

Recall:

Labels: donor, acceptor, N-type, P-type, Conduction band, Valence band,  $E_g$

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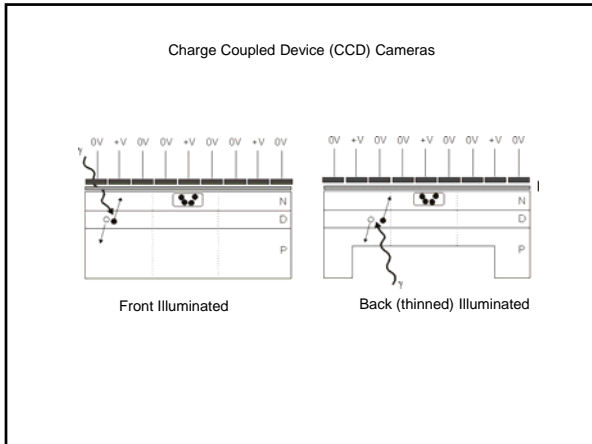
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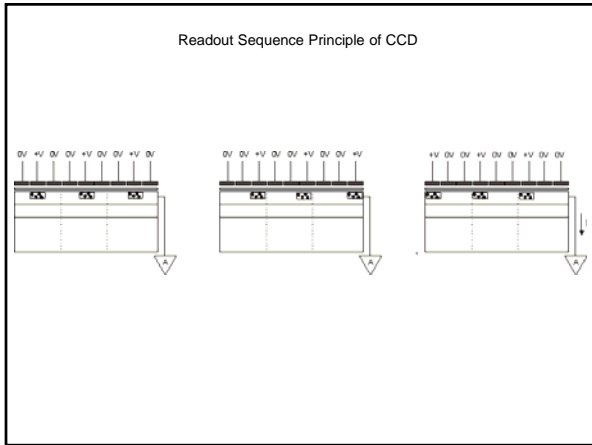
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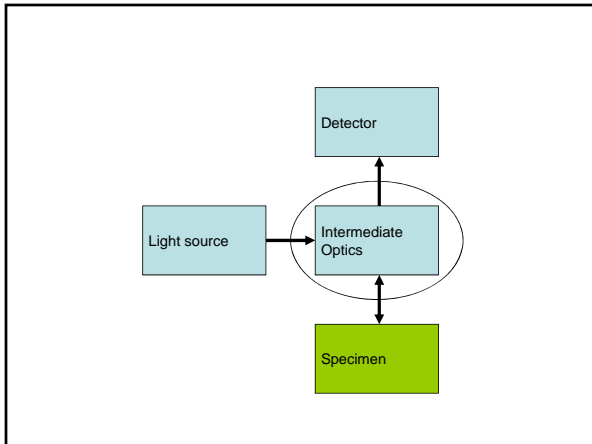
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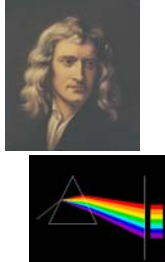
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Wave and Particle Nature of Light

Wave Nature of Light -- Huygen



Particle Nature of Light -- Newton



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Physical Optics – Wave nature of light

Maxwell and His Equations



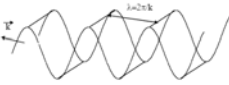
$$\begin{aligned} \nabla \cdot \vec{E} &= 4\pi\rho \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} &= 0 \\ \nabla \times \vec{B} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} &= \frac{4\pi}{c} \vec{j} \end{aligned}$$

Wave Equations

$$\begin{aligned} \nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} &= 0 \\ \nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} &= 0 \end{aligned}$$

Plane Wave Solution

$$\vec{E}(x,t) = \vec{E}_0 \cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \quad ck = \omega$$


Plane wave propagates like a "ray" of light

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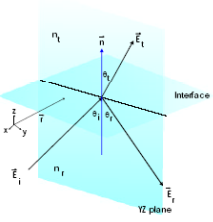
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Reflection and Refraction of Light at Boundary



Reflection

$$\sin \theta_i = \sin \theta_r$$

Refraction (Snell's Law)

$$n_i \sin \theta_i = n_t \sin \theta_t$$

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**Simple Ray Tracing I**

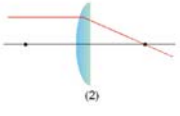
Ray Tracing is just based on the application of Snell's law to a curved (spherical) surface. We will focus on 4 simple rules of ray tracing.

Rays pass through the focal point becomes parallel to the optical axis.  
Rays parallel to the optical axis are deflected through the focal point.



(1)

Rays originated from the focal point emerge parallel to the optical axis



(2)

Rays parallel to the optical axis converges to the focal point

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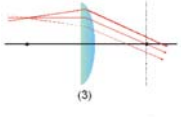
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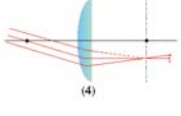
**Simple Ray Tracing II**

Rays originate from the focal plane becomes collimated.  
Collimated rays converges at the focal plane.



(3)

Rays originated from the plane emerge collimated



(4)

Collimated rays emerge focus at The focal plane

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**Optical element I: Lens**

We have been using lens throughout this lecture, it may be useful to pause for a moment to describe what are the typical terminology associated with lens.

 <b>Biconvex</b> $R1 > 0$ $R2 < 0$	 <b>Biconcave</b> $R1 < 0$ $R2 > 0$
 <b>Plano-convex</b> $R1 = \infty$ $R2 < 0$	 <b>Plano-concave</b> $R1 = \infty$ $R2 > 0$
 <b>Meniscus convex</b> $R1 > 0$ $R2 > 0$	 <b>Meniscus concave</b> $R1 < 0$ $R2 < 0$

**Optical element II: mirrors, prism, apertures**

These are common optical elements that is mostly self explanatory and I will not spend much time on these.

**Mirrors:** Mirrors has similar terminology as lens but only has one surface.

**Prisms:** Prisms has a number of applications such as dispersing different color of light and directing light and image.

**Apertures & Stops:** As discussed before, aperture and stops serves to define optical path and to minimize aberration effect by eliminating non-paraxial rays.

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Optical Microscopy I

Detection path of an optical microscope. Note that at the detector, the magnification is the ratio of the focal length of the objective and the tube lens.

Magnification  
 $M = \frac{x'}{x} = \frac{f_2}{f_1}$

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Optical Microscopy III

Kohler illumination ensure that the structure of the light source (such as the filament of lamp) is not imaged at the specimen.

(A)                      (B)

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Microscopic contrast and resolution

Two of the most important and difficult to quantify aspects of an optical microscope are its ability to generate contrast and its ability to resolve fine structures

What is contrast? Contrast refers to an "intensity" difference between a specimen of interest and its background.

Optically contrast is defined as the visibility:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

What is resolution? Resolution defines how fine we can see ... how far apart two objects have to be for them to be distinguishable.

Rayleigh's Criterion:

Two objects are distinguishable if their centers are separation by *greater than their full width at half maximum*

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### Diffraction III

In the far field (Fraunhofer) limit,  $R \gg D$

We can approximate  $r$  by  $R$ , and the distance  $r$  can be approximated as a function of  $R$ ,  $y$  and  $\theta$  (using Law of cosine):

$$r = R - y \sin \theta + \left(\frac{y^2}{2R}\right) \cos^2 \theta + \dots$$

Keeping terms to first order in  $y$ , we have

$$dE = \frac{E}{R} \sin(\omega t - kR + ky \sin \theta)$$

The total field at  $P$  is:

$$E = \frac{E}{R} \int_{-D/2}^{D/2} \sin(\omega t - kR + ky \sin \theta) dy$$

Therefore, we have

$$E = \frac{ED}{R} \frac{\sin[(kD/2) \sin \theta]}{(kD/2) \sin \theta} \sin(\omega t - kR)$$


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### Diffraction IV

Defining  $\beta = (kD/2) \sin \theta$  and calculating the intensity at  $P$  we have:

$$I(\theta) = \frac{1}{2} \left(\frac{ED}{R}\right)^2 \left(\frac{\sin \beta}{\beta}\right)^2 = I(0) \text{sinc}^2(\beta)$$

Note that sinc function corresponds to a number of fringes reflecting the fundamental interference effect of diffraction.

The maxima and minima location of the intensity can be identify by the first and 2<sup>nd</sup> derivatives of  $I(\theta)$ :

$$\frac{dI}{d\beta} = I(0) \frac{2 \sin \beta (\beta \cos \beta - \sin \beta)}{\beta^3} = 0$$

From the 2<sup>nd</sup> derivative, we get the minima is corresponding to the solution of

$$\sin \beta = 0$$

The maxima corresponds to the solutions of:

$$\tan \beta = \beta$$

Note that in this derivation, we have ignored near field effects (Fresnel diffraction) as well as the vector nature of the electric field.

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### Diffraction V

$$I(\theta) = I(0) \left[ \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]^2$$

Microscopy imaging can be consider as the diffraction from a circular aperture with a lens for focusing – diffraction results in "broadening" of the focal point.

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**Interference I**

Consider combining two plane waves:

$$\vec{E}_1(\vec{r}, t) = \vec{E}_{01} \cos(\vec{k}_1 \cdot \vec{r} - \omega t)$$

$$\vec{E}_2(\vec{r}, t) = \vec{E}_{02} \cos(\vec{k}_2 \cdot \vec{r} - \omega t) \quad \left| \vec{k}_1 \right| = \left| \vec{k}_2 \right| = k$$

The combined field is

$$\vec{E}(\vec{r}, t) = \vec{E}_{01} \cos(\vec{k}_1 \cdot \vec{r} - \omega t) + \vec{E}_{02} \cos(\vec{k}_2 \cdot \vec{r} - \omega t)$$

The brightness or intensity is the "mean square" of the field

$$I = \frac{1}{T} \int_0^T \vec{E}(\vec{r}, t) \cdot \vec{E}^*(\vec{r}, t) dt \equiv \langle E^2 \rangle$$

$$= \frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + 2\sqrt{\frac{E_{01}^2}{2} \frac{E_{02}^2}{2}} \cos[(\vec{k}_1 - \vec{k}_2) \cdot \vec{r}]$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$\delta$  is a phase factor measuring the path length difference of the two beams at  $\vec{r}$  multiplied by  $k$

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**Fourier Optics I**

Recall the interference of two plane waves

$$\vec{E}_1(\vec{r}, t) = \vec{E} \cos(\vec{k}_1 \cdot \vec{r} + \omega t)$$

$$\vec{E}_2(\vec{r}, t) = \vec{E} \cos(\vec{k}_2 \cdot \vec{r} + \omega t)$$

$$I(\vec{r}, t) = 2I + 2I \cos(\vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r})$$

In the case where the waves incident symmetrically and looking at the intensity along the y axis

$$\vec{k}_1 = k \sin \theta \hat{x} + k \cos \theta \hat{y}$$

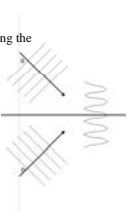
$$\vec{k}_2 = k \sin \theta \hat{x} - k \cos \theta \hat{y}$$

$$\vec{r} = y \hat{y}$$

The intensity has a simple distribution depend on angle  $\theta$ :

$$I(\vec{r}, t) = 2I(1 + \cos(2k \cos \theta y))$$

Note that when angle is zero degree (light wave counter propagating), the highest frequency oscillation is observed at spatial frequency:  $2k = 2\pi(\frac{2}{\lambda})$ . When the waves are parallel, angle is 90 degree, the spatial frequency is zero (constant intensity light).




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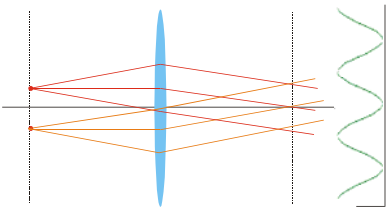
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**Fourier Optics II**

Consider two point source at the focal plane of a lens, the light rays become collimated plane waves after the lens and interference is observed.




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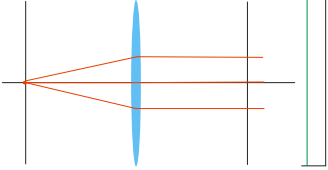
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**Fourier Optics III**

What happen when the two sources coincide? Only parallel plane waves are generated.



The diagram shows a lens (blue) between two vertical planes. On the left plane, two points are marked with red dots. Red lines represent light rays originating from these points and passing through the lens. On the right plane, the rays emerge as parallel horizontal lines, representing plane waves.

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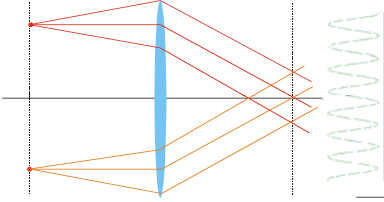
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**Fourier Optics IV**

What happen if the point sources are made further apart?



The diagram shows a lens (blue) between two vertical planes. On the left plane, two points are marked with red dots, spaced further apart than in the previous diagram. Red lines represent light rays originating from these points and passing through the lens. On the right plane, the rays emerge as a complex wave pattern with multiple peaks and troughs, representing a higher spatial frequency component.

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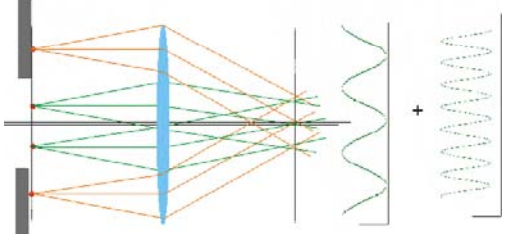
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**Resolution viewed from Fourier Optics**

Light emission from any object in the specimen plane can be Decomposed into its Fourier components. Which Fourier component will pass the finite aperture of the objective lens? Low frequencies!



The diagram shows a specimen plane (left) with a grey bar representing an object. Light rays (red and green) pass through a lens (blue) and are focused onto a Fourier plane (middle). The Fourier plane shows two wave patterns: a low-frequency wave (green) and a high-frequency wave (red). A plus sign (+) is between the two wave patterns, indicating their superposition.

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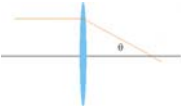
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Resolution viewed from Fourier Optics II



$$NA = n \sin(\theta)$$

What is the maximum frequency that can be pass? Consider the case of a very large lens (numerical aperture, NA approach one). The waves will approach counter propagating and the maximum frequency is:

$$k_{\max} = 2\pi\left(\frac{2}{\lambda}\right)$$

Note that maximum spatial frequency is a function of wavelength. Shorter wavelength implies higher frequency (resolution) imaging.

At a given wavelength, we should expect a resolution of about  $\frac{\lambda}{2}$

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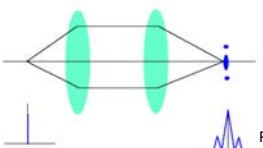
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Resolution viewed from Fourier Optics III



Point Spread function

More quantitative analysis shows that:  $I(\theta) = I(0) \left[ \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]^2$

$$J_1(ka \sin \theta) = J_1\left(\frac{2\pi}{\lambda} a \frac{r}{f}\right) = J_1\left(\frac{2\pi}{\lambda} \frac{a}{f} r\right) = J_1\left(\frac{2\pi}{\lambda} NA r\right)$$

$$J_1(x) = 0 \text{ at } x = 3.83 \Rightarrow r_{\min} = \frac{0.6\lambda}{NA}$$


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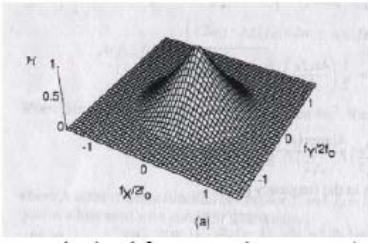
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Resolution viewed from Fourier Optics VI

$$OTF(k) = F(PSF(r))$$



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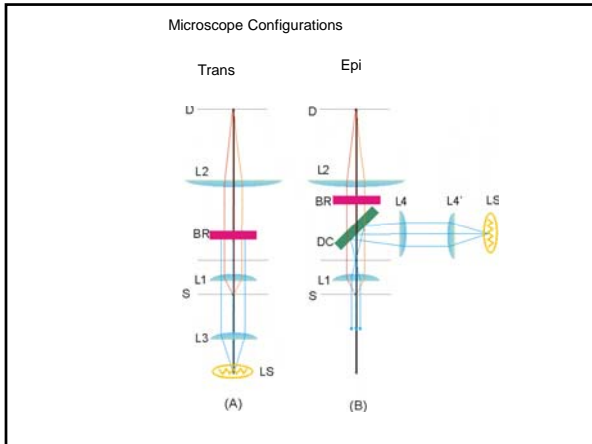
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Fluorescence is fundamentally a quantum phenomena

Light ray can be thought of as a stream of photons each having energy:

$$E = h\nu = h \frac{c}{\lambda}$$

Light-Molecule Interaction

Polyatomic molecules

Simple orbitals

S P

Jablonski Diagram

excitation

fluorescence

phosphorescence

intersystem crossing

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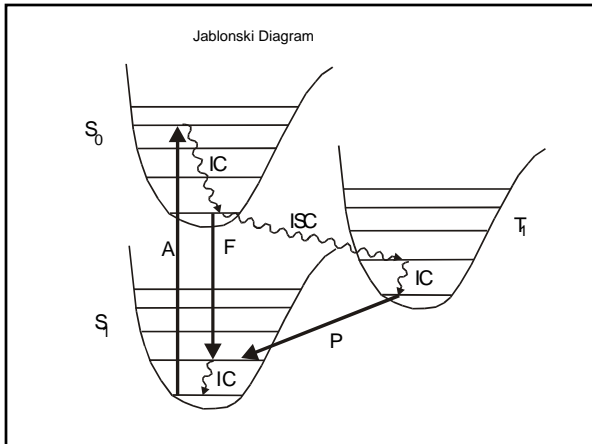
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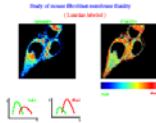
**Basic Fluorescence Measurement**

*Intensity measurement.*

This is the most basic measurement. It is not very diagnostic and it mainly reflects the presence or absence of fluorophore and their concentrations. Note that quantitative intensity measurement is very hard as many factors affect the excited state of the fluorophore and will modify its intensity especially in biological systems.

*Spectral measurement.*

Spectral measurement is quite diagnostic. Most fluorophores has a fairly unique spectral pattern. Spectral measurement allows the experimenter to determine what fluorophores are present. In microscopy setting, the interaction of microscopic structures can be studied if they can be labeled with different color fluorophores. Equally important, many fluorophores changes color (excited state vibrational level shifts) as a function of biochemical environment. This allows a sensitive monitoring of intracellular or tissue biochemical state. The calcium probe described earlier is a good example. Another example is this membrane probe Laurdan which changes color as a function of the fluidity of the membrane.




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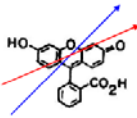
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Emission spectra is defined as measuring emission intensity as a function of wavelength at a given excitation wavelength. Excitation spectra is defined as the measurement of emission intensity at a given emission wavelength as a function of excitation wavelength.

*Polarization and Isotropy*

Polarization is also another useful property of fluorescence. All fluorescence molecules have a preferential direction of excitation (excitation dipole) and emission (emission dipole). Note that the excitation and emission dipoles do not have to coincide in general. The probability of exciting a molecule depends on the relative orientation of the molecular excitation dipole and the polarization of light. Let  $\theta$  be the angle between the light polarization and the molecule excitation dipole. The probability of excitation is:  $P \propto \cos^2 \theta$ . This is similar to what we see for the transmission of a polarizer. One can also see that exciting molecules with polarized light selects a sub-population of molecule that are oriented close the polarization of light.




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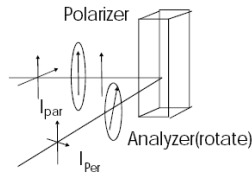
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The measurement of polarization of aqueous specimen is typically performed using the above geometry. Excitation light is first polarized. The emission light is analyzed for its polarization parallel and perpendicular to the excitation direction.

The result is expressed in terms of polarization, P, or anisotropy, r:

$$P = \frac{I_{par} - I_{per}}{I_{par} + I_{per}}, \quad r = \frac{I_{par} - I_{per}}{I_{par} + 2I_{per}}$$

Note that the steady state polarization is high with rotation diffusion rate slow compared with its lifetime but its polarization is low if diffusion is fast compared with its lifetime. This is very useful for measuring the binding of small ligand to large molecules or surfaces. Polarization is also often used to measure the mean orientation of molecules.

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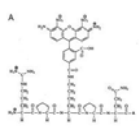
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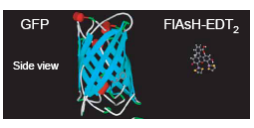
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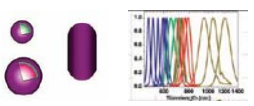
Fluorescent Probes



Molecular Probes, Oregon



GFP FIASH-EDT<sub>2</sub>  
Side view  
Hoffmann et al, Nat. Meth, 2005



Michalet et al, Science, 2005

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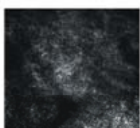
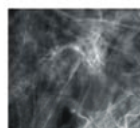
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Strengths of Fluorescence Microscopy

(1) New contrast enhancement mechanism



Imaging collagen/elastin fibers in dermis. Fluorescence image (left), scattered light image (right)

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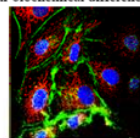
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Strengths of Fluorescence Microscopy

(2) Specificity – individual structural components can be tagged based on their biochemical difference



Nuclei (blue) is label with DAPI, Actin (green) is label with Bodipy phalloidin, mitochondria (red) is label with MitoTracker.

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Strengths of Fluorescence Microscopy

(3) Molecular Sensitivity

Yildiz, Acc. Chem. Res., 2005

Alexandrakis, Nat. Med., 2004

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Strengths of Fluorescence Microscopy

(4) Image biochemical reactions/ Monitor microenvironmental changes

Fan et al. Biophys. J., 1999  
Calcium wave in HeLa cells

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Strengths of Fluorescence Microscopy

(5) Monitors genetic expression

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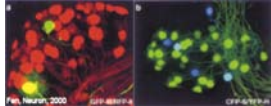
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**The Need For 3D Resolved Imaging**

Biological systems are inherently 3D!

Biological processes also occur on multiple length scale

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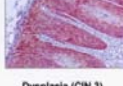
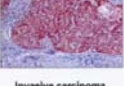
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**Histopathology**

Normal proliferation	Inflammation	Metaplasia
		
		
Dysplasia (CIN 1,2)	Dysplasia (CIN 3)	Invasive carcinoma

Solution: mechanical sectioning of specimen

Comment: (1) Clinical standard (2) Simple technology  
(3) Sectioning artifacts (4) Not in vivo

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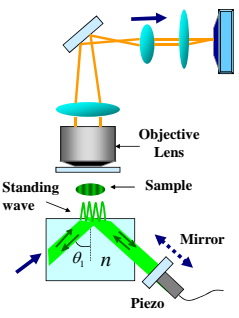
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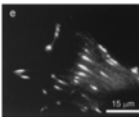
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**Total Internal Reflection Microscopy**

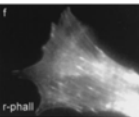


Solution: Evanescence wave at interfaces

Comments: (1) only basal surface structure (2) high z resolution, 50 nm



TIRF



Wide Field

Sund & Alexrod, Biophys J 2000

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True 3D Microscopy

Confocal Microscopy: Minsky, US Patent, 1961

Two-Photon Microscopy: Sheppard et al., IEEE J of QE, 1977  
Denk et al., Science, 1990

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The Invention of Confocal Microscopy

Confocal microscopy is invented by Prof. Melvin Minsky of MIT in about 1950s.

**United States Patent Office** 3,013,467  
Patented Dec. 19, 1961

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3,013,467  
MICROSCOPY APPARATUS  
Melvin Minsky, 44 Cambridge St., Cambridge, Mass.  
Filed Nov. 7, 1957, Ser. No. 475,187  
4 Claims. (Cl. 348-144)

My invention relates to a new and improved electronic microscope apparatus and to a novel apparatus for microscopy.

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can in an identical manner and can produce identical results.

FIG. 1 shows, by way of illustration, a light source 10 which may be in the form of an electron light bulb or other light source suitable for use in microscopy. The bulb 10 has the usual reflector 12.

Mounted in front of the light source 10 is a focusing device, which, in an simplest form may be a glass or wall 14 having a pinhole aperture 14 in registry with the light source 10. The structure described above is shown in

Dec. 19, 1961      M. MINSKY      3,013,467  
MICROSCOPY APPARATUS  
Filed Nov. 7, 1957

FIG. 1

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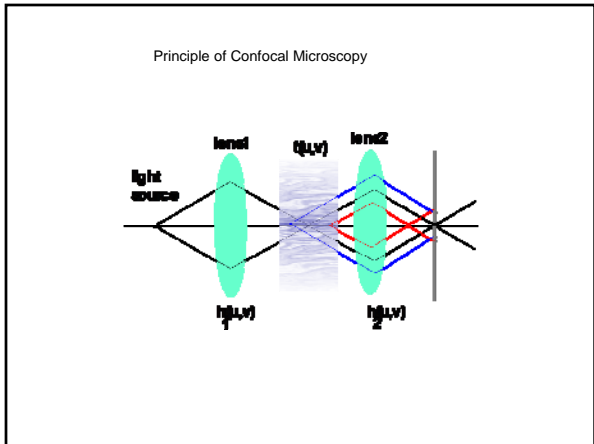
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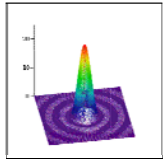
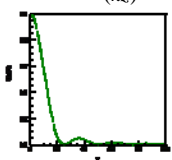
**Point Spread Function of Confocal Microscopy**

Lateral Dimension: Airy function

$$PSF_{confocal}(kr) \propto \left[ \frac{2J_1(kr)}{kr} \right]^4$$

k is the wave number

Axial Dimension : Sinc function

$$PSF_{confocal}(kz) \propto \left[ \frac{\sin(kz)}{kz} \right]^4$$



Resolution:

Lateral  $\propto NA$

Axial  $\propto NA^2$

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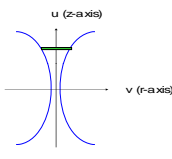
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Depth discrimination

For a uniform specimen, we can ask how much fluorescence is generated at each z-section above and below the focal plane assuming that negligible amount of light is absorbed throughout.



$$F_{z=sec}(u) \equiv 2\pi \int_0^{\infty} F(u,v) v dv$$


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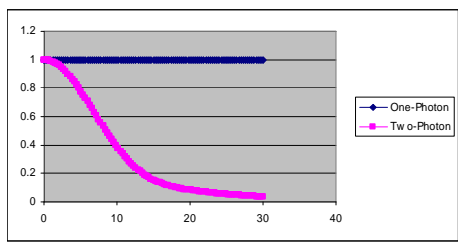
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Depth discrimination




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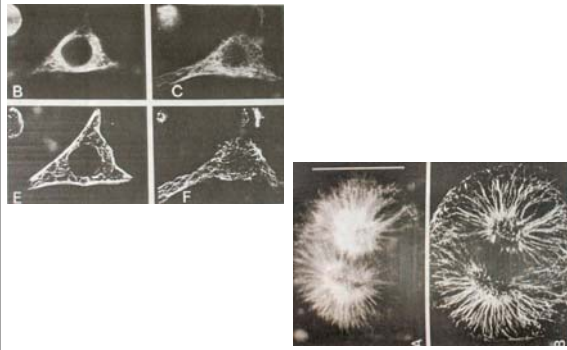
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Early Demonstration of Confocal Microscopy in Biological Imaging



White et al., JCB 1987

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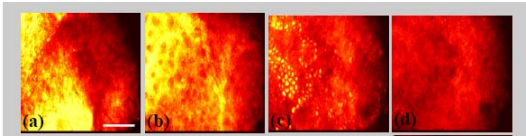
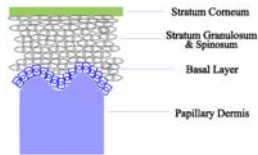
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Some Recent Application of Confocal Tissue Imaging



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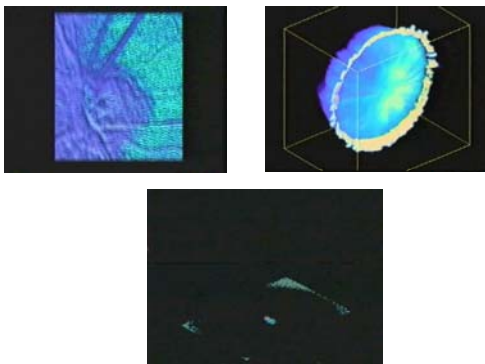
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Confocal Tissue Imaging



Masters, Opt Lett, 1999

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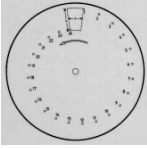
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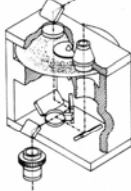
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**Tandem Scanning Confocal Microscope**


Utilizes a Nipkow Disk



Holes organize in an Archimedes spiral



Petran's System



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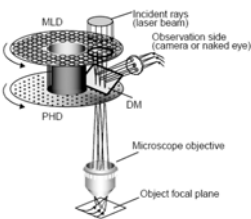
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**A Model Tandem Confocal Microscope Utilizing Yokogawa Scan Head**



Eliminate light throughput Issue by spinning both a plate of lenslets and other plate of pinholes

0/30 sec	1/30 sec	2/30 sec	3/30 sec	4/30 sec
5/30 sec	6/30 sec	7/30 sec	8/30 sec	9/30 sec
10/30 sec	11/30 sec	12/30 sec	13/30 sec	14/30 sec
15/30 sec	16/30 sec	17/30 sec	18/30 sec	19/30 sec

**C. Elegans**

0 sec	0.1 sec	0.2 sec	0.3 sec
0.4 sec	0.5 sec	0.6 sec	0.7 sec

Calcium events in nerve fiber

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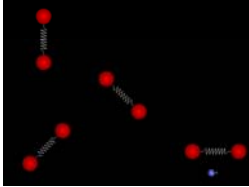
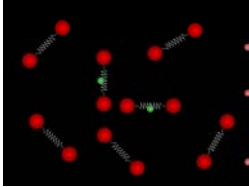
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Basic Ideas of two-photon excitation

Two-photon excitation is similar to the one-photon process except the use of two lower energy (infrared) photons.

The difference between the two can be seen in these movies:

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An abbreviated history of two-photon microscopy

- (1930) Maria Goeppert-Mayer predicted the existence of two-photon effect
- (1961) Franken et al. demonstrated second harmonic generation in ruby
- (1961) Kaiser et al. showed two-photon fluorescence in a solid state material
- (1964) Singh and Bradley reports three-photon fluorescence
- (1970s-1980s) Two-photon effect has been used in biological spectroscopy by researchers such as Birge, Fredrich, and McCain
- (1970s-1980s) Microanalysis based on non-linear 2nd harmonic generation was developed by researchers such as Freund and Hellwarth
- (1976-80s) A number of researchers such as Sheppard, Kompfner, Gannaway and Wilson suggested the possibility of incorporating non-linear excitation into scanning microscopy
- (1989) Denk, Webb and coworkers definitively demonstrated two-photon scanning microscopy and a number of its unique properties such as the triggering of localized chemical reaction

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Two-Photon Excitation Microscopy

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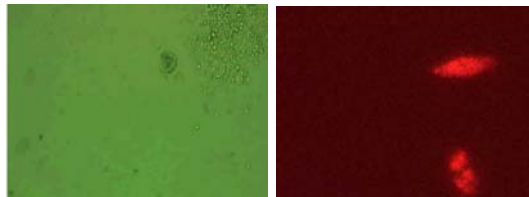
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A comparison of two-photon and confocal microscopes

- (1) Confocal microscopes have better resolution than two-photon microscopes without confocal detection.
- (2) Two-photon microscope results in less photodamage in biological specimens. The seminal work by the White group in U. Wisconsin on the development of *c. elegans* and hamsters provides some of the best demonstration. After embryos have been continuously imaged for over hours, live specimens are born after implantation.




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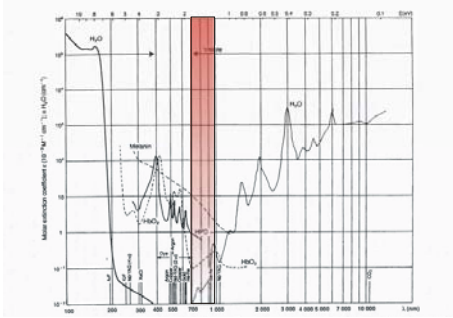
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(3) Two-photon microscope provides better penetration into highly scattering tissue specimen. Infrared light has lower absorption and lower scattering in turbid media.




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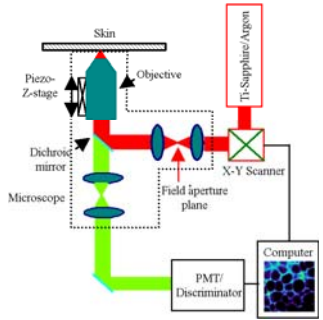
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The design of a two-photon microscope

Two-photon microscope design is actually significantly simpler than that of confocal microscope and has much in common.




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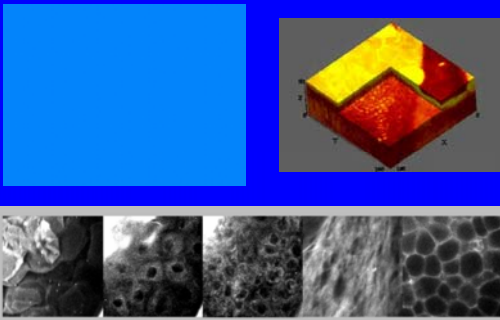
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**A 3D Reconstructed Movie Of Skin Structures From A Mouse Ear Tissue Punch**



In collaboration with I. Kochevar, Wellman Labs, MGH and B. Masters

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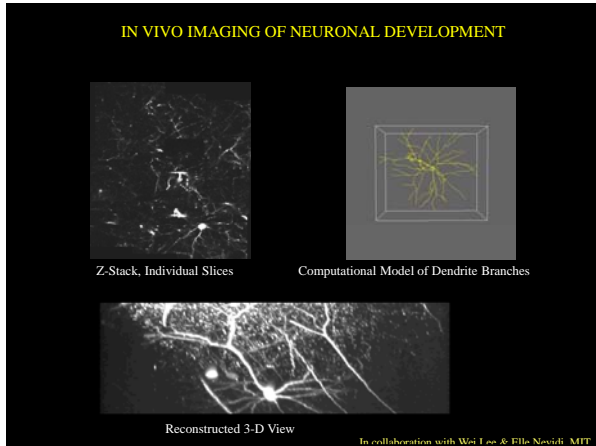
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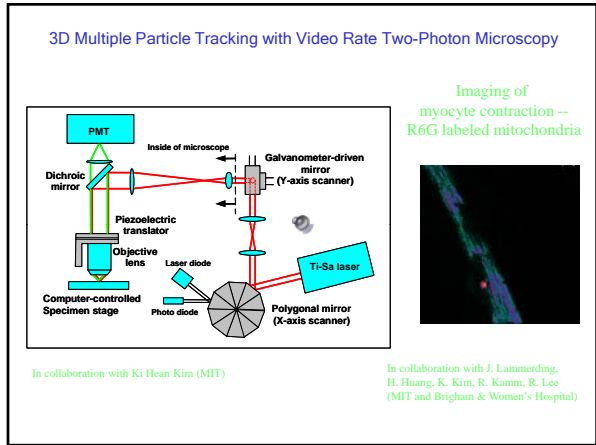
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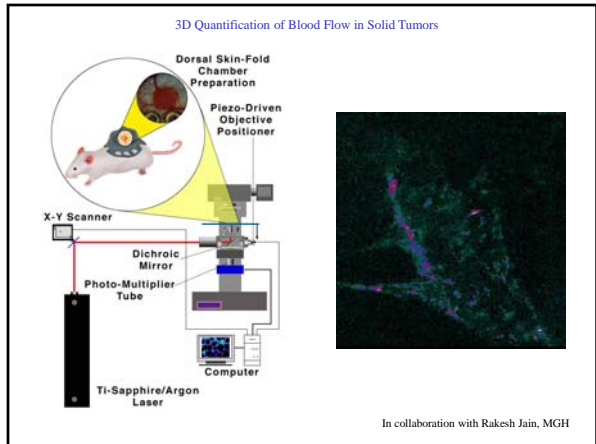
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### MMM Ray Tracing

2-Photon Microscope – multi foci scanning

Multi-foci

Resonant-mirror-position

Fixed micro-lens array for Resonant-mirror setup

In collaboration with Karsten Bahlmann (MIT), Mike Kirber (BU Medical Center)

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### MAPMT-based MMM

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### Spontaneous contraction of cardiac myocyte cells labeled with fluo3

White light

800 Images at 640 frames/sec : 1,25 sec sequence

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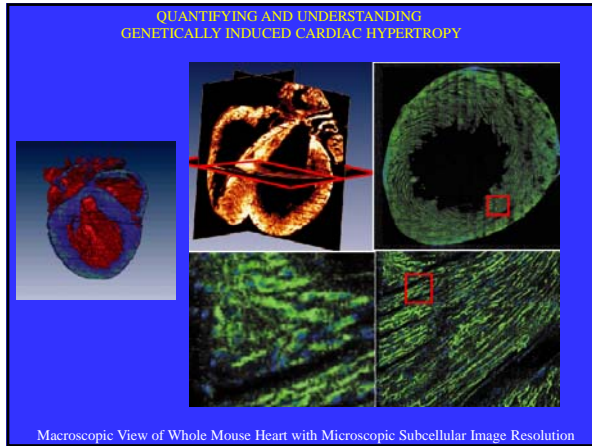
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