

Lecture 7

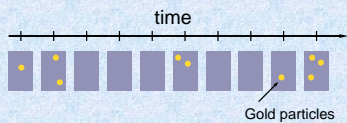
FCS, Autocorrelation, PCH, Cross-correlation

Joachim Mueller

Principles of Fluorescence Techniques
Laboratory for Fluorescence Dynamics

Figure and slide acknowledgements:
Enrico Gratton

Historic Experiment: 1st Application of Correlation Spectroscopy (Svedberg & Inouye, 1911) *Occupancy Fluctuation*



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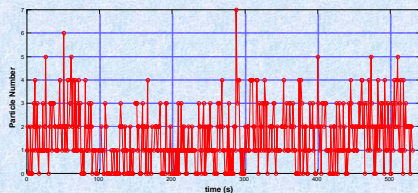
Svedberg and Inouye, *Zeitschr. F. physik. Chemie* 1911, 77-145

Collected data by counting (by visual inspection) the number of particles in the observation volume as a function of time using a "ultra microscope"

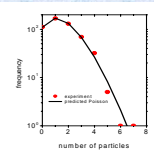
💡 Statistical analysis of raw data required

Particle Correlation

Graph of Raw Data:

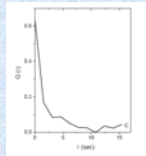


*Histogram of particle counts



• Poisson statistics
 $\langle N \rangle = 1.55$

*Autocorrelation



• Autocorrelation not available in the original paper. It can be easily calculated today.

$$\langle N \rangle = \frac{1}{G(0)} = 1.56$$

What we learn from the correlation function?

Svedberg claimed: Gold colloids with radius $R = 3 \text{ nm}$ $D_{\text{expected}} = \frac{k_B T}{6\pi\eta R} \approx 70 \frac{\mu\text{m}^2}{\text{s}}$
(Stokes-Einstein)

Experimental facts:

Slit \longleftrightarrow
 $2 \mu\text{m}$

$\langle \Delta x^2 \rangle \approx 2D\tau_D$

characteristic diffusion time
 $\tau_D \approx 1.5 \text{ s}$

$\Rightarrow D_{\text{Exp}} \approx 1 \frac{\mu\text{m}^2}{\text{s}}$

$\Rightarrow R \approx 200 \text{ nm}$

Conclusion: Bad sample preparation

The ultramicroscope was invented in 1903 (*Siedentopf and Zsigmondy*). They already concluded that scattering will not be suitable to observe single molecules, but **fluorescence** could.

Fluorescence Correlation Spectroscopy (FCS)

In FCS
Fluctuations are in the Fluorescence Signal

Diffusion

Enzymatic Activity

Phase Fluctuations

Conformational Dynamics

Rotational Motion

Protein Folding

Example of processes that could generate fluctuations

Generating Fluctuations By Motion

Sample

Fluorescence

Observation Volume

Coverslip

objective

What is Observed?

1. The Rate of Motion
2. The Concentration of Particles
3. Changes in the Particle Fluorescence while under Observation, for example conformational transitions

Autocorrelation Function

$$G(\tau) = \frac{\langle \delta F(t) \delta F(t + \tau) \rangle}{\langle F(t) \rangle^2}$$

Factors influencing the fluorescence signal:

$$F(t) = \kappa Q \int d\mathbf{r} W(\mathbf{r}) C(\mathbf{r}, t)$$

κQ = quantum yield and detector sensitivity (how bright is our probe). This term could contain the fluctuation of the fluorescence intensity due to internal processes
 $W(\mathbf{r})$ describes our observation volume
 $C(\mathbf{r}, t)$ is a function of the fluorophore concentration over time. This is the term that contains the "physics" of the diffusion processes

Average fluorescence signal: $\langle F(t) \rangle$

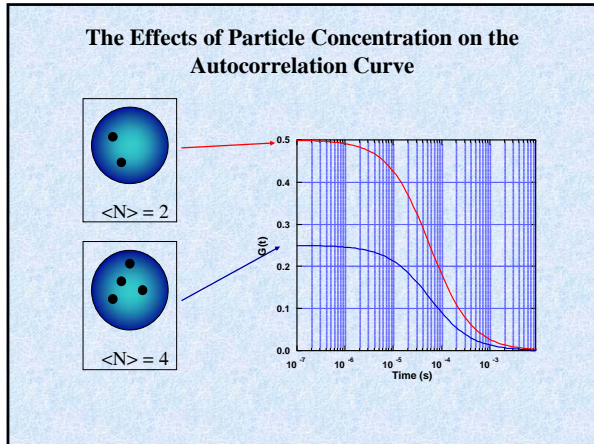
Fluorescence fluctuation: $\delta F(t) = F(t) - \langle F(t) \rangle$

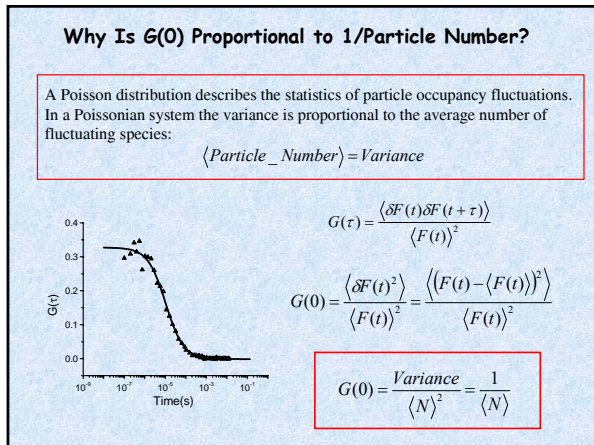
Calculating the Autocorrelation Function

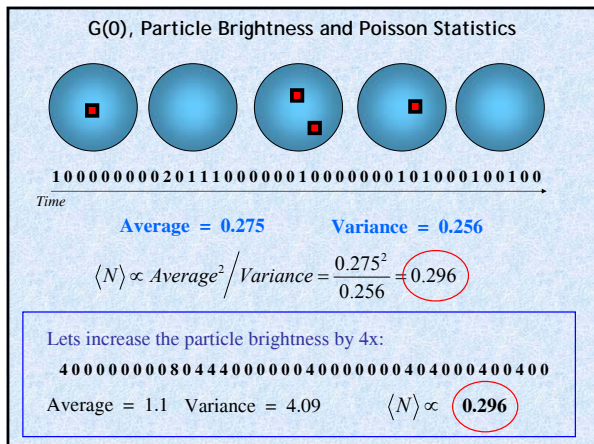
$$G(\tau) = \frac{\langle \delta F(t) \cdot \delta F(t + \tau) \rangle}{\langle F \rangle^2}$$

The Autocorrelation Function

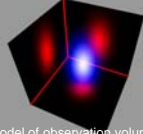
$$G(\tau) = \frac{\langle \delta F(t) \delta F(t + \tau) \rangle}{\langle F(t) \rangle^2}$$







**Effect of Shape on the (2-Photon)
Autocorrelation Functions:
(for simple diffusion)**



Model of observation volume

For a 3-dimensional Gaussian excitation volume:

$$G(\tau) = \frac{\gamma}{N} \left(1 + \frac{8D\tau}{w_0^2} \right)^{-1} \left(1 + \frac{8D\tau}{z_0^2} \right)^{-1/2}$$

For a 2-dimensional Gaussian excitation volume:

$$G(\tau) = \frac{\gamma}{N} \left(1 + \frac{8D\tau}{w_0^2} \right)^{-1}$$

- γ : shape factor (0.354 for 3DG, 0.5 for 2DG)
- N : average number of particles inside volume
- D : Diffusion coefficient
- w_0 : radial beam waist of two-photon laser spot
- z_0 : axial beam waist of two-photon laser spot

1-photon equation contains a 4, instead of 8

Additional Equations:

3D Gaussian Confocor analysis:

$$G(\tau) = 1 + \frac{1}{N} \left(1 + \frac{\tau}{\tau_D} \right)^{-1} \cdot \left(1 + S^2 \cdot \frac{\tau}{\tau_D} \right)^{-1/2}$$

... where N is the average particle number, τ_D is the diffusion time (related to D , $\tau_D = w^2/8D$ for two photon and $\tau_D = w^2/4D$ for 1-photon excitation), and S is a shape parameter, equivalent to w/z in the previous equations.

Note: The offset of one is caused by a different definition of $G(\tau)$: $G(\tau) = \frac{\langle F(t+\tau) \cdot F(t) \rangle}{\langle F \rangle^2}$

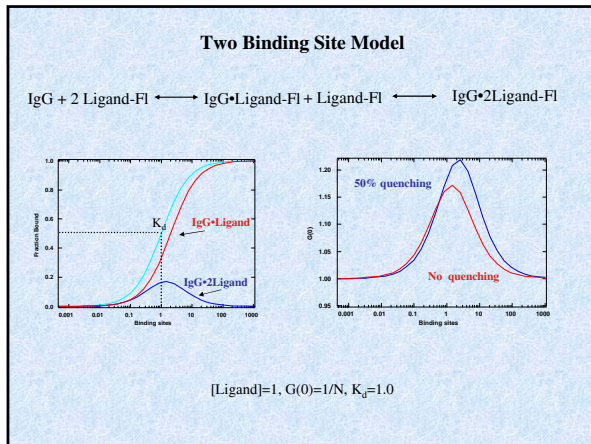
Triplet state term:

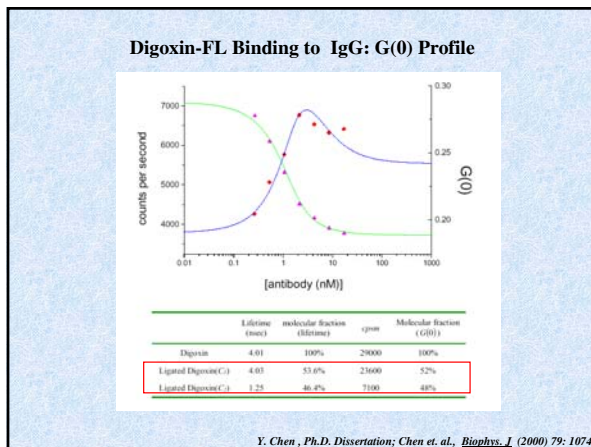
$$\left(1 + \frac{T}{1-T} e^{-\frac{\tau}{\tau_T}} \right)$$

...where T is the triplet state amplitude and τ_T is the triplet lifetime.

Orders of magnitude (for 10nM solution, small molecule, water)

Volume	Device	Size(μm)	Molecules	Diffusion Time (s)
milliliter	cuvette	10000	6×10^{12}	10^4
microliter	plate well	1000	6×10^9	10^2
nanoliter	microfabrication	100	6×10^6	1
picoliter	typical cell	10	6×10^3	10^{-2}
femtoliter	confocal volume	1	6×10^0	10^{-4}
attoliter	nanofabrication	0.1	6×10^{-3}	10^{-6}





Multiple Species

Case 2: Species vary by a difference in brightness

assuming that $D_1 \approx D_2$

The quantity $G(0)$ becomes the only parameter to distinguish species,
but we know that:


$$G(0)_{\text{sample}} = \sum f_i^2 \cdot G(0)_i$$

The autocorrelation function is not suitable
for analysis of this kind of data without additional information.

We need a different type of analysis

Photon Counting Histogram (PCH)

Aim: To resolve species from differences in their molecular brightness



Poisson Distribution for particle number: $p(N) = \frac{\langle N \rangle^N \cdot e^{-\langle N \rangle}}{N!}$

But distribution of photon counts is Non-Poissonian: $p(k) = PCH(\epsilon, \langle N \rangle)$

Single Species: $p(k)$ is the probability of observing k photon counts

Sources of Non-Poissonian Noise

- Detector Noise
- Diffusing Particles in an Inhomogeneous Excitation Beam*
- Particle Number Fluctuations*
- Multiple Species*

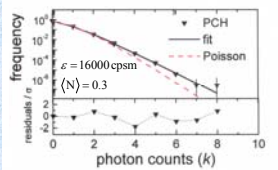
Photon Counting Histogram (PCH)

Aim: To resolve species from differences in their molecular brightness

Molecular brightness ϵ : The average photon count rate of a single fluorophore

PCH: probability distribution function $p(k)$

where $p(k)$ is the probability of observing k photon counts

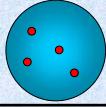


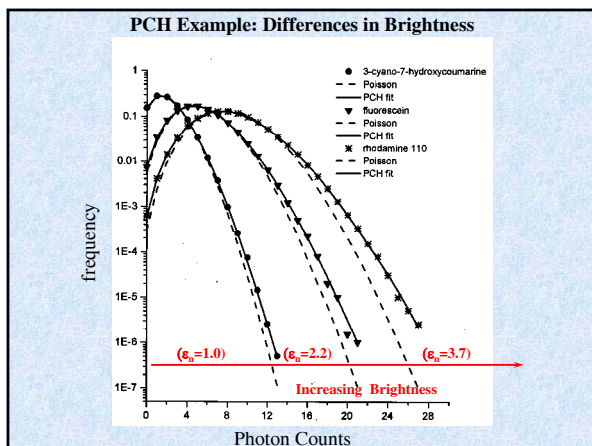
Single Species: $p(k) = PCH(\epsilon, \langle N \rangle)$

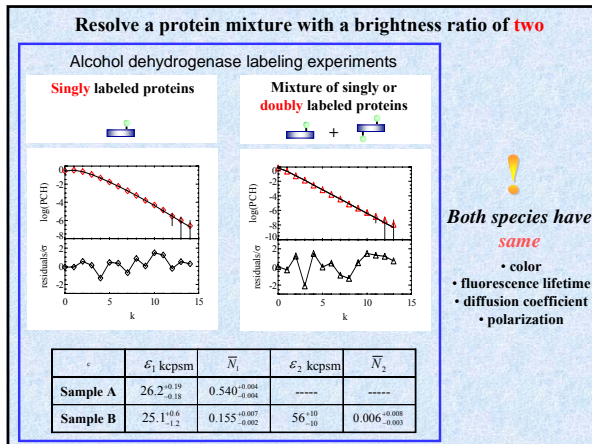
Note: PCH is Non-Poissonian!

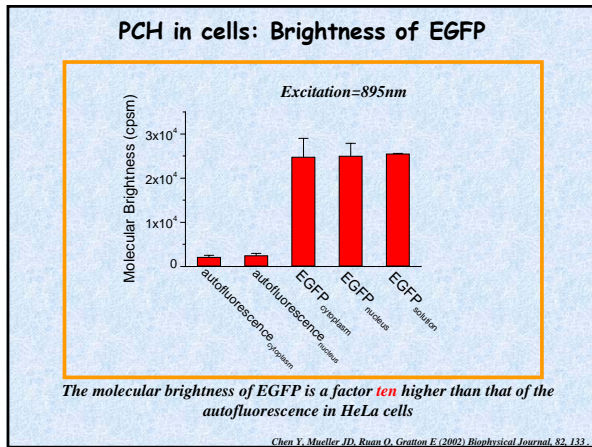
Sources of Non-Poissonian Noise

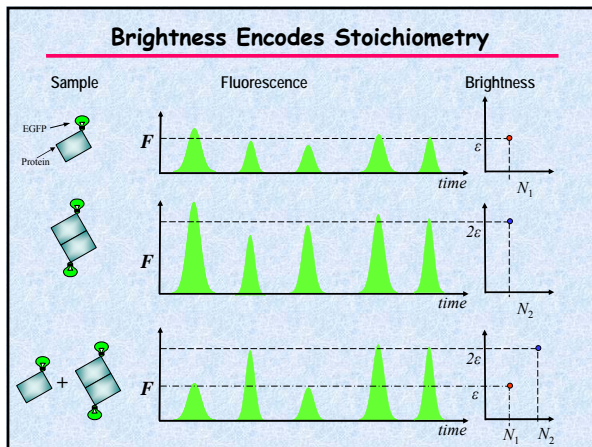
- Detector Noise
- Diffusing Particles in an Inhomogeneous Excitation Beam*
- Particle Number Fluctuations*
- Multiple Species*

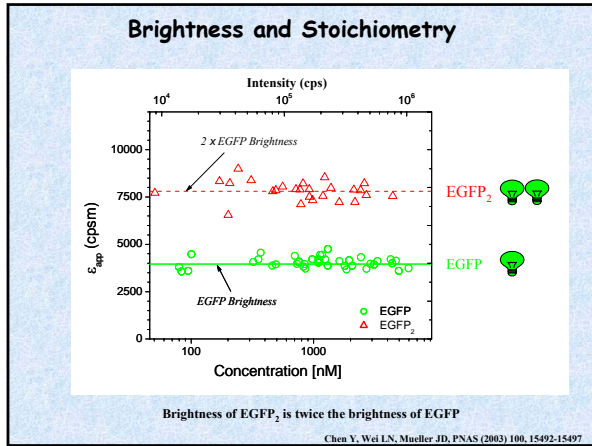


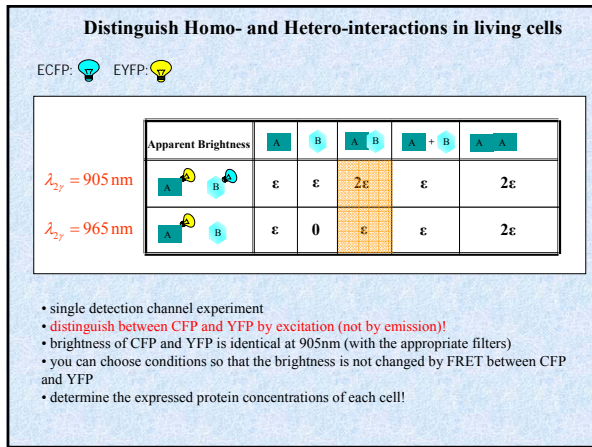


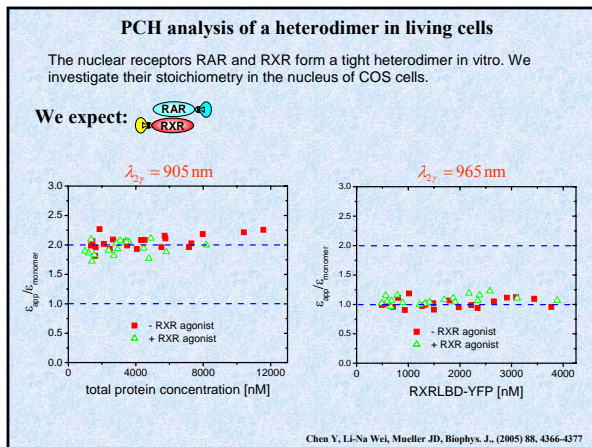


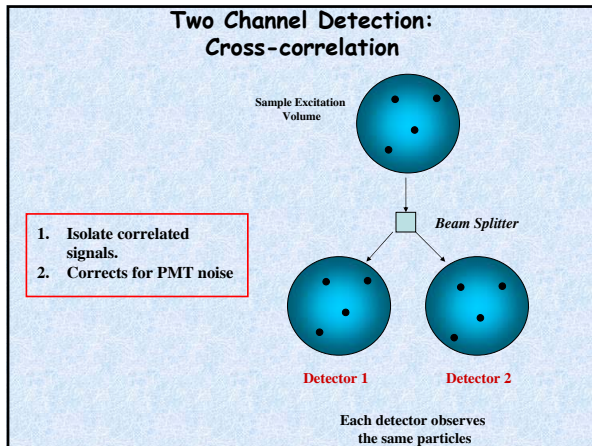


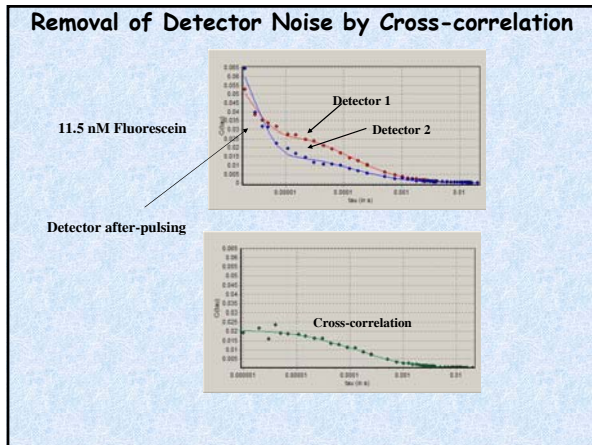


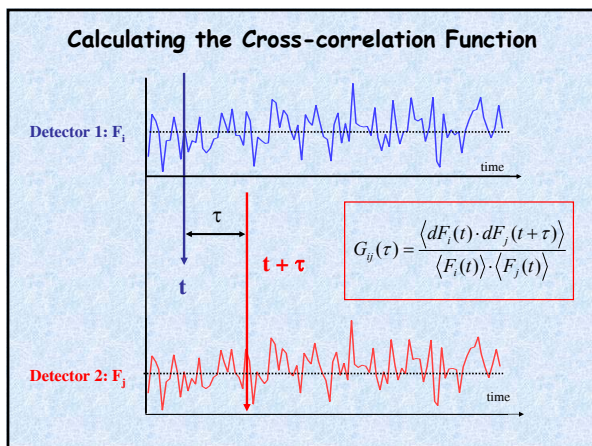












Cross-correlation Calculations

One uses the same fitting functions you would use for the standard autocorrelation curves.

Thus, for a 3-dimensional Gaussian excitation volume one uses:

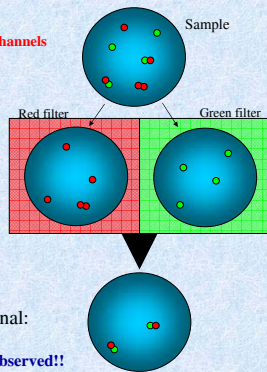
$$G_{12}(\tau) = \frac{\gamma}{N_{12}} \left(1 + \frac{8D_{12}\tau}{w^2} \right)^{-1} \left(1 + \frac{8D_{12}\tau}{z^2} \right)^{-1/2}$$

G_{12} is commonly used to denote the cross-correlation and G_1 and G_2 for the autocorrelation of the individual detectors. Sometimes you will see $G_x(0)$ or $C(0)$ used for the cross-correlation.

Two-Color Cross-correlation

The cross-correlation
ONLY if particles are observed in both channels

Each detector observes particles with a particular color



The cross-correlation signal:
Only the green-red molecules are observed!!

Experimental Concerns: Excitation Focusing & Emission Collection

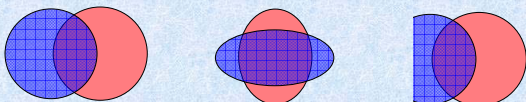
We assume exact match of the observation volumes in our calculations which is difficult to obtain experimentally.

Excitation side:

- (1) Laser alignment
- (2) Chromatic aberration
- (3) Spherical aberration

Emission side:

- (1) Chromatic aberrations
- (2) Spherical aberrations
- (3) Improper alignment of detectors or pinhole
(cropping of the beam and focal point position)



Two-color Cross-correlation

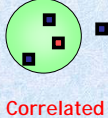
Equations are similar to those for the cross correlation using a simple beam splitter:

$$G_{ij}(\tau) = \frac{\langle dF_i(t) \cdot dF_j(t + \tau) \rangle}{\langle F_i(t) \rangle \cdot \langle F_j(t) \rangle}$$

Information Content	Signal
Correlated signal from particles having both colors .	$G_{12}(\tau)$
Autocorrelation from channel 1 on the green particles .	$G_1(\tau)$
Autocorrelation from channel 2 on the red particles .	$G_2(\tau)$

Spectral Crosstalk

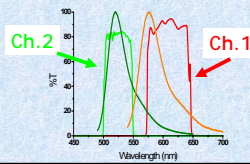
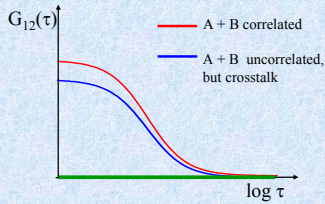
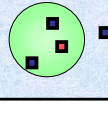
Uncorrelated



Correlated

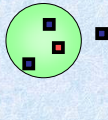


Uncorrelated+Crosstalk

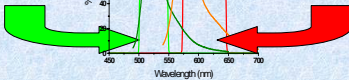


Two-Color FCS: Correct for Spectral Overlap

Uncorrelated



$$F_2(t) = f_{12}N_1 + f_{22}N_2$$

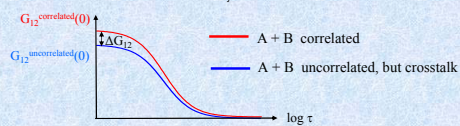


$$F_1(t) = f_{11}N_1 + f_{21}N_2$$

For two uncorrelated species, the amplitude of the cross-correlation is proportional to:

$$G_{12}^{\text{uncorrelated}}(0) \propto \frac{f_{11}f_{12}\langle N_1 \rangle + f_{11}f_{22}\langle N_2 \rangle}{f_{11}f_{12}\langle N_1 \rangle^2 + (f_{11}f_{22} + f_{21}f_{12})\langle N_1 \rangle\langle N_2 \rangle + f_{21}f_{22}\langle N_2 \rangle^2}$$

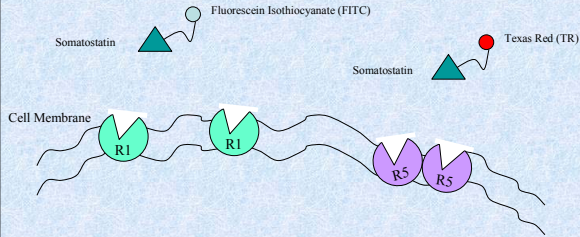
f_{ij} : fractional intensity of species i in channel j



Does SSTR1 exist as a monomer after ligand binding while SSTR5 exists as a dimer/oligomer?

Collaboration with Ramesh Patel^{†1} and Ujendra Kumar^{*}

[†]Fraser Laboratories, Departments of Medicine, Pharmacology, and Therapeutics and Neurology and Neurosurgery, McGill University, and Royal Victoria Hospital, Montreal, QC, Canada H3A 2T4; ^{*}Department of Chemistry and Physics, Clarkson University, Potsdam, NY 13699

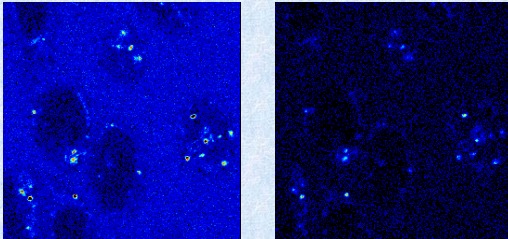


Three Different CHO-K1 cell lines: wt R1, HA-R5, and wt R1/HA-R5
Hypothesis: R1 - monomer ; R5 - dimer/oligomer; R1R5 dimer/oligomer

SSTR1 CHO-K1 cells with SST-fitc + SST-tr

Green Ch.

Red Ch.



- Very little labeled SST inside cell nucleus
- Non-homogeneous distribution of SST
- Impossible to distinguish co-localization from molecular interaction

